

The Legacy of Leonhard Euler

A Tricentennial Tribute

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \zeta(s), \quad \zeta(1) = \sum_{n=1}^{\infty} \frac{1}{n}, \quad \zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

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