

Surveys of Modern Mathematics
Volume I

Analytic Methods in Algebraic Geometry

by Jean-Pierre Demailly

Contents

Introduction	1
Chapter 1. Preliminary Material: Cohomology, Currents	5
1.A. Dolbeault Cohomology and Sheaf Cohomology	5
1.B. Plurisubharmonic Functions	6
1.C. Positive Currents	9
Chapter 2. Lelong numbers and Intersection Theory	15
2.A. Multiplication of Currents and Monge-Ampère Operators	15
2.B. Lelong Numbers	18
Chapter 3. Hermitian Vector Bundles, Connections and Curvature	25
Chapter 4. Bochner Technique and Vanishing Theorems	31
4.A. Laplace-Beltrami Operators and Hodge Theory	31
4.B. Serre Duality Theorem	32
4.C. Bochner-Kodaira-Nakano Identity on Kähler Manifolds	33
4.D. Vanishing Theorems	34
Chapter 5. L^2 Estimates and Existence Theorems	37
5.A. Basic L^2 Existence Theorems	37
5.B. Multiplier Ideal Sheaves and Nadel Vanishing Theorem	39
Chapter 6. Numerically Effective and Pseudo-effective Line Bundles	47
6.A. Pseudo-effective Line Bundles and Metrics with Minimal Singularities	47
6.B. Nef Line Bundles	49
6.C. Description of the Positive Cones	51
6.D. The Kawamata-Viehweg Vanishing Theorem	56
6.E. A Uniform Global Generation Property due to Y.T. Siu	58
Chapter 7. A Simple Algebraic Approach to Fujita's Conjecture	61
Chapter 8. Holomorphic Morse Inequalities	71
8.A. General Analytic Statement on Compact Complex Manifolds	71
8.B. Algebraic Counterparts of the Holomorphic Morse Inequalities	72
8.C. Asymptotic Cohomology Groups	74
8.D. Transcendental Asymptotic Cohomology Functions	78
Chapter 9. Effective Version of Matsusaka's Big Theorem	83
Chapter 10. Positivity Concepts for Vector Bundles	89
Chapter 11. Skoda's L^2 Estimates for Surjective Bundle Morphisms	99
11.A. Surjectivity and Division Theorems	99
11.B. Applications to Local Algebra: the Briançon-Skoda Theorem	105

Chapter 12. The Ohsawa-Takegoshi L^2 Extension Theorem	111
12.A. The Basic a Priori Inequality	111
12.B. Abstract L^2 Existence Theorem for Solutions of $\bar{\partial}$ -Equations	112
12.C. The L^2 Extension Theorem	114
12.D. Skoda's Division Theorem for Ideals of Holomorphic Functions	122
Chapter 13. Approximation of Closed Positive Currents by Analytic Cycles	127
13.A. Approximation of Plurisubharmonic Functions Via Bergman kernels	127
13.B. Global Approximation of Closed $(1,1)$ -Currents on a Compact Complex Manifold	129
13.C. Global Approximation by Divisors	136
13.D. Singularity Exponents and log Canonical Thresholds	143
13.E. Hodge Conjecture and approximation of (p,p) -currents	148
Chapter 14. Subadditivity of Multiplier Ideals and Fujita's Approximate Zariski Decomposition	153
Chapter 15. Hard Lefschetz Theorem with Multiplier Ideal Sheaves	159
15.A. A Bundle Valued Hard Lefschetz Theorem	159
15.B. Equisingular Approximations of Quasi Plurisubharmonic Functions	160
15.C. A Bochner Type Inequality	166
15.D. Proof of Theorem 15.1	168
15.E. A Counterexample	170
Chapter 16. Invariance of Plurigenera of Projective Varieties	173
Chapter 17. Numerical Characterization of the Kähler Cone	177
17.A. Positive Classes in Intermediate (p,p) -bidegrees	177
17.B. Numerically Positive Classes of Type $(1,1)$	178
17.C. Deformations of Compact Kähler Manifolds	184
Chapter 18. Structure of the Pseudo-effective Cone and Mobile Intersection Theory	189
18.A. Classes of Mobile Curves and of Mobile $(n-1, n-1)$ -currents	189
18.B. Zariski Decomposition and Mobile Intersections	192
18.C. The Orthogonality Estimate	199
18.D. Dual of the Pseudo-effective Cone	202
18.E. A Volume Formula for Algebraic $(1,1)$ -Classes on Projective Surfaces	205
Chapter 19. Super-canonical Metrics and Abundance	209
19.A. Construction of Super-canonical Metrics	209
19.B. Invariance of Plurigenera and Positivity of Curvature of Super-canonical Metrics	216
19.C. Tsuji's Strategy for Studying Abundance	217
Chapter 20. Siu's Analytic Approach and Păun's Non Vanishing Theorem	219
References	223