How Euler Did it

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How Euler Did It is a collection of 40 monthly columns that appeared on MAA Online between November 2003 and February 2007 about the mathematical and scientific work of the great 18th-century Swiss mathematician Leonhard Euler. Almost every column is self-contained and gives the context, significance and some of the details of a particular facet of his work.

First we find interesting stories about Euler's work in geometry. In a discussion of the Euler polyhedral formula, the author speculates about whether Descartes had a role in Euler's discovery and analyzes the flaw in Euler's proof. We also learn of Euler's solution to Cramer's paradox and its role in the early days of linear algebra.

Number theory is well represented. We see Euler's first proof of Fermat's little theorem for which he used mathematical induction, as well as his discovery of over a hundred pairs of amicable numbers, and his work on odd perfect numbers, about which little is known even today.

Elsewhere in the book we learn of the development of what we now call Venn diagrams, what Euler knew about orthogonal matrices, Euler's ideas on the foundations of calculus (before the days of limits, epsilons and deltas), and his proof that mixed partial derivatives are equal.

Professor Sandifer based his columns on Euler's own words in the original languages in which they were written. In this way, the author was able to uncover many details that are not found in other sources. For example, we see how Euler used differential equations and continued fractions to prove that the constant e is irrational, several years before Lambert, who is usually credited with this discovery. Euler also made an observation equivalent to saying that the number of primes less than a number x is approximately $x/\ln x$, an observation usually attributed to Gauss some 15 years after Euler died.

The collection ends with a somewhat playful, but factual, account of Euler's role in the discovery of America.