

Turbulent Flows

Stephen B. Pope

Cornell University

Contents

<i>List of tables</i>	<i>page</i> xv
<i>Preface</i>	xvii
<i>Nomenclature</i>	xxi
PART ONE: FUNDAMENTALS	1
1 Introduction	3
1.1 The nature of turbulent flows	3
1.2 The study of turbulent flows	7
2 The equations of fluid motion	10
2.1 Continuum fluid properties	10
2.2 Eulerian and Lagrangian fields	12
2.3 The continuity equation	14
2.4 The momentum equation	16
2.5 The role of pressure	18
2.6 Conserved passive scalars	21
2.7 The vorticity equation	22
2.8 Rates of strain and rotation	23
2.9 Transformation properties	24
3 The statistical description of turbulent flows	34
3.1 The random nature of turbulence	34
3.2 Characterization of random variables	37
3.3 Examples of probability distributions	43
3.4 Joint random variables	54
3.5 Normal and joint-normal distributions	61
3.6 Random processes	65
3.7 Random fields	74
3.8 Probability and averaging	79

4	Mean-flow equations	83
4.1	Reynolds equations	83
4.2	Reynolds stresses	86
4.3	The mean scalar equation	91
4.4	Gradient-diffusion and turbulent-viscosity hypotheses	92
5	Free shear flows	96
5.1	The round jet: experimental observations	96
5.1.1	A description of the flow	96
5.1.2	The mean velocity field	97
5.1.3	Reynolds stresses	105
5.2	The round jet: mean momentum	111
5.2.1	Boundary-layer equations	111
5.2.2	Flow rates of mass, momentum, and energy	115
5.2.3	Self-similarity	116
5.2.4	Uniform turbulent viscosity	118
5.3	The round jet: kinetic energy	122
5.4	Other self-similar flows	134
5.4.1	The plane jet	134
5.4.2	The plane mixing layer	139
5.4.3	The plane wake	147
5.4.4	The axisymmetric wake	151
5.4.5	Homogeneous shear flow	154
5.4.6	Grid turbulence	158
5.5	Further observations	161
5.5.1	A conserved scalar	161
5.5.2	Intermittency	167
5.5.3	PDFs and higher moments	173
5.5.4	Large-scale turbulent motion	178
6	The scales of turbulent motion	182
6.1	The energy cascade and Kolmogorov hypotheses	182
6.1.1	The energy cascade	183
6.1.2	The Kolmogorov hypotheses	184
6.1.3	The energy spectrum	188
6.1.4	Restatement of the Kolmogorov hypotheses	189
6.2	Structure functions	191
6.3	Two-point correlation	195
6.4	Fourier modes	207
6.4.1	Fourier-series representation	207
6.4.2	The evolution of Fourier modes	211

6.4.3	The kinetic energy of Fourier modes	215
6.5	Velocity spectra	219
6.5.1	Definitions and properties	220
6.5.2	Kolmogorov spectra	229
6.5.3	A model spectrum	232
6.5.4	Dissipation spectra	234
6.5.5	The inertial subrange	238
6.5.6	The energy-containing range	240
6.5.7	Effects of the Reynolds number	242
6.5.8	The shear-stress spectrum	246
6.6	The spectral view of the energy cascade	249
6.7	Limitations, shortcomings, and refinements	254
6.7.1	The Reynolds number	254
6.7.2	Higher-order statistics	255
6.7.3	Internal intermittency	258
6.7.4	Refined similarity hypotheses	260
6.7.5	Closing remarks	263
7	Wall flows	264
7.1	Channel flow	264
7.1.1	A description of the flow	264
7.1.2	The balance of mean forces	266
7.1.3	The near-wall shear stress	268
7.1.4	Mean velocity profiles	271
7.1.5	The friction law and the Reynolds number	278
7.1.6	Reynolds stresses	281
7.1.7	Lengthscales and the mixing length	288
7.2	Pipe flow	290
7.2.1	The friction law for smooth pipes	290
7.2.2	Wall roughness	295
7.3	Boundary layers	298
7.3.1	A description of the flow	299
7.3.2	Mean-momentum equations	300
7.3.3	Mean velocity profiles	302
7.3.4	The overlap region reconsidered	308
7.3.5	Reynolds-stress balances	313
7.3.6	Additional effects	320
7.4	Turbulent structures	322

PART TWO: MODELLING AND SIMULATION	333
8 An introduction to modelling and simulation	335
8.1 The challenge	335
8.2 An overview of approaches	336
8.3 Criteria for appraising models	336
9 Direct numerical simulation	344
9.1 Homogeneous turbulence	344
9.1.1 Pseudo-spectral methods	344
9.1.2 The computational cost	346
9.1.3 Artificial modifications and incomplete resolution	352
9.2 Inhomogeneous flows	353
9.2.1 Channel flow	353
9.2.2 Free shear flows	354
9.2.3 Flow over a backward-facing step	355
9.3 Discussion	356
10 Turbulent-viscosity models	358
10.1 The turbulent-viscosity hypothesis	359
10.1.1 The intrinsic assumption	359
10.1.2 The specific assumption	364
10.2 Algebraic models	365
10.2.1 Uniform turbulent viscosity	365
10.2.2 The mixing-length model	366
10.3 Turbulent-kinetic-energy models	369
10.4 The k - ε model	373
10.4.1 An overview	373
10.4.2 The model equation for ε	375
10.4.3 Discussion	382
10.5 Further turbulent-viscosity models	383
10.5.1 The k - ω model	383
10.5.2 The Spalart–Allmaras model	385
11 Reynolds-stress and related models	387
11.1 Introduction	387
11.2 The pressure–rate-of-strain tensor	388
11.3 Return-to-isotropy models	392
11.3.1 Rotta’s model	392
11.3.2 The characterization of Reynolds-stress anisotropy	393
11.3.3 Nonlinear return-to-isotropy models	398
11.4 Rapid-distortion theory	404
11.4.1 Rapid-distortion equations	405

11.4.2	The evolution of a Fourier mode	406
11.4.3	The evolution of the spectrum	411
11.4.4	Rapid distortion of initially isotropic turbulence	415
11.4.5	Final remarks	421
11.5	Pressure–rate-of-strain models	422
11.5.1	The basic model (LRR-IP)	423
11.5.2	Other pressure–rate-of-strain models	425
11.6	Extension to inhomogeneous flows	428
11.6.1	Redistribution	428
11.6.2	Reynolds-stress transport	429
11.6.3	The dissipation equation	432
11.7	Near-wall treatments	433
11.7.1	Near-wall effects	433
11.7.2	Turbulent viscosity	434
11.7.3	Model equations for k and ε	435
11.7.4	The dissipation tensor	436
11.7.5	Fluctuating pressure	439
11.7.6	Wall functions	442
11.8	Elliptic relaxation models	445
11.9	Algebraic stress and nonlinear viscosity models	448
11.9.1	Algebraic stress models	448
11.9.2	Nonlinear turbulent viscosity	452
11.10	Discussion	457
12	PDF methods	463
12.1	The Eulerian PDF of velocity	464
12.1.1	Definitions and properties	464
12.1.2	The PDF transport equation	465
12.1.3	The PDF of the fluctuating velocity	467
12.2	The model velocity PDF equation	468
12.2.1	The generalized Langevin model	469
12.2.2	The evolution of the PDF	470
12.2.3	Corresponding Reynolds-stress models	475
12.2.4	Eulerian and Lagrangian modelling approaches	479
12.2.5	Relationships between Lagrangian and Eulerian PDFs	480
12.3	Langevin equations	483
12.3.1	Stationary isotropic turbulence	484
12.3.2	The generalized Langevin model	489
12.4	Turbulent dispersion	494

12.5	The velocity–frequency joint PDF	506
12.5.1	Complete PDF closure	506
12.5.2	The log-normal model for the turbulence frequency	507
12.5.3	The gamma-distribution model	511
12.5.4	The model joint PDF equation	514
12.6	The Lagrangian particle method	516
12.6.1	Fluid and particle systems	516
12.6.2	Corresponding equations	519
12.6.3	Estimation of means	523
12.6.4	Summary	526
12.7	Extensions	529
12.7.1	Wall functions	529
12.7.2	The near-wall elliptic-relaxation model	534
12.7.3	The wavevector model	540
12.7.4	Mixing and reaction	545
12.8	Discussion	555
13	Large-eddy simulation	558
13.1	Introduction	558
13.2	Filtering	561
13.2.1	The general definition	561
13.2.2	Filtering in one dimension	562
13.2.3	Spectral representation	565
13.2.4	The filtered energy spectrum	568
13.2.5	The resolution of filtered fields	571
13.2.6	Filtering in three dimensions	575
13.2.7	The filtered rate of strain	578
13.3	Filtered conservation equations	581
13.3.1	Conservation of momentum	581
13.3.2	Decomposition of the residual stress	582
13.3.3	Conservation of energy	585
13.4	The Smagorinsky model	587
13.4.1	The definition of the model	587
13.4.2	Behavior in the inertial subrange	587
13.4.3	The Smagorinsky filter	590
13.4.4	Limiting behaviors	594
13.4.5	Near-wall resolution	598
13.4.6	Tests of model performance	601
13.5	LES in wavenumber space	604
13.5.1	Filtered equations	604

13.5.2	Triad interactions	606
13.5.3	The spectral energy balance	609
13.5.4	The spectral eddy viscosity	610
13.5.5	Backscatter	611
13.5.6	A statistical view of LES	612
13.5.7	Resolution and modelling	615
13.6	Further residual-stress models	619
13.6.1	The dynamic model	619
13.6.2	Mixed models and variants	627
13.6.3	Transport-equation models	629
13.6.4	Implicit numerical filters	631
13.6.5	Near-wall treatments	634
13.7	Discussion	635
13.7.1	An appraisal of LES	635
13.7.2	Final perspectives	638
PART THREE: APPENDICES		641
<i>Appendix A Cartesian tensors</i>		643
A.1	Cartesian coordinates and vectors	643
A.2	The definition of Cartesian tensors	647
A.3	Tensor operations	649
A.4	The vector cross product	654
A.5	A summary of Cartesian-tensor suffix notation	659
<i>Appendix B Properties of second-order tensors</i>		661
<i>Appendix C Dirac delta functions</i>		670
C.1	The definition of $\delta(x)$	670
C.2	Properties of $\delta(x)$	672
C.3	Derivatives of $\delta(x)$	673
C.4	Taylor series	675
C.5	The Heaviside function	675
C.6	Multiple dimensions	677
<i>Appendix D Fourier transforms</i>		678
<i>Appendix E Spectral representation of stationary random processes</i>		683
E.1	Fourier series	683
E.2	Periodic random processes	686
E.3	Non-periodic random processes	689
E.4	Derivatives of the process	690
<i>Appendix F The discrete Fourier transform</i>		692

<i>Appendix G</i>	Power-law spectra	696
<i>Appendix H</i>	Derivation of Eulerian PDF equations	702
<i>Appendix I</i>	Characteristic functions	707
<i>Appendix J</i>	Diffusion processes	713
	<i>Bibliography</i>	727
	<i>Author index</i>	749
	<i>Subject index</i>	754