Abstract

The book is first of all a history of category theory from the beginnings to A. Grothendieck and F.W. Lawvere. Category theory was an important conceptual tool in 20th century mathematics whose influence on some mathematical subdisciplines (above all algebraic topology and algebraic geometry) is analyzed. Category theory also has an important philosophical aspect: on the one hand its set-theoretical foundation is less obvious than for other mathematical theories, and on the other hand it unifies conceptually a large part of modern mathematics and may therefore be considered as somewhat fundamental itself. The role of this philosophical aspect in the historical development is the second focus of the book. Relying on the historical analysis, the author develops a philosophical interpretation of the theory of his own, intending to get closer to how mathematicians conceive the significance of their activity than traditional schools of philosophy of science.
The book is the first monography exclusively devoted to the history of category theory. To a substantial extent it considers aspects never studied before. The author uses (and justifies the use of) a methodology combining historical and philosophical approaches. The analysis is not confined to general remarks, but goes into considerable mathematical detail. Hence, the book provides an exceptionally thorough case study compared with other works on history or philosophy of mathematics. The philosophical position developed here (inspired by Peircean pragmatism and Wittgenstein) is an interesting alternative to traditional approaches in philosophy of mathematics like platonism, formalism and intuitionism.

Inhalt

*Introduction: The subject matter of the present book - Secondary literature and sources - Some remarks concerning historical methodology
Prelude: Poincaré, Wittgenstein, Peirce, and the use of concepts: A plea for philosophy of mathematics - Using concepts - Reductionist vs. pragmatist epistemology of mathematics
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Category theory in Homological Algebra: Homological algebra for modules - Development of the sheaf concept until 1957 - The Tôhoku paper - Conclusions
Category theory in Algebraic Geometry: Conceptual innovations by Grothendieck - The Weil conjectures - Grothendieck's methodology and categories
From tool to object: full-fledged category theory: Some concepts transformed in categorial language - Important steps in the theory of functors - What is the concept of object about? - Categories as objects of study
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Categorial foundations: The concept of foundation of mathematics - Lawvere's categorial foundations: a historical overview - Elementary toposes and "local foundations Categorial foundations and foundational problems of CT - General objections, in particular the argument of "psychological priority"
Pragmatism and category theory: Category theorists and category theory - Which epistemology for mathematics?