

Abstract

Euler is one of the greatest and most prolific mathematicians of all time. He wrote the first accessible books on calculus, created the theory of circular functions, and discovered new areas of research such as elliptic integrals, the calculus of variations, graph theory, divergent series, and so on. It took hundreds of years for his successors to develop in full the theories he began, and some of his themes are still at the center of today's mathematics. It is of great interest therefore to examine his work and its relation to current mathematics. This book attempts to do that.

In number theory the discoveries he made empirically would require for their eventual understanding such sophisticated developments as the reciprocity laws and class field theory. His pioneering work on elliptic integrals is the precursor of the modern theory of abelian functions and abelian integrals. His evaluation of zeta and multizeta values is not only a fantastic and exciting story but very relevant to us, because they are at the confluence of much research in algebraic geometry and number theory today.

Anticipating his successors by more than a century, Euler created a theory of summation of series that do not converge in the traditional manner. Chapter 5 of the book treats the progression of ideas regarding divergent series from Euler to many parts of modern analysis and quantum physics.

The last chapter contains a brief treatment of Euler products. Euler discovered the product formula over the primes for the zeta function as well as for a small number of what are now called Dirichlet L-functions. Here the book goes into the development of the theory of such Euler products and the role they play in number theory, thus offering the reader a glimpse of current developments (the Langlands program).

Inhalt

*Leonhard Euler (1707-1783): Early life - The first stay in St. Petersburg: 1727-1741 - The Berlin years: 1741-1766 - The second St. Petersburg stay and the last years: 1766-1783 - Opera Omnia - The personality of Euler
The Universal Mathematician: Calculus - Elliptic integrals - Calculus of variations - Number theory

Zeta Values: Some remarks on infinite series and products and their values - Evaluation of $\zeta(2)$ and $\zeta(4)$ - Infinite products for circular and hyperbolic functions - The infinite partial fractions for $(\sin x)^{-1}$ and $\cot x$. Evaluation of $\zeta(2k)$ and $L(2k + 1)$ - Partial fraction expansions as integrals - Multizeta values

Euler-Maclaurin Sum Formula: Formal derivation - The case when the function is a polynomial - Summation formula with remainder terms - Applications
Divergent Series and Integrals: Divergent series and Euler's ideas about summing them - Euler's derivation of the functional equation of the zeta function - Euler's summation of the factorial series - The general theory of summation of divergent series - Borel summation - Tauberian theorems - Some applications - Fourier integral, Wiener Tauberian theorem, and Gel'fand transform on commutative Banach algebras - Generalized functions and smeared summation - Gaussian integrals, Wiener measure and the path integral formulae of Feynman and Kac

Euler Products: Euler's product formula for the zeta function and others - Euler products from Dirichlet to Hecke - Euler products from Ramanujan and Hecke to Langlands - Abelian extensions and class field theory - Artin nonabelian L-functions - The Langlands program