

## Abstract

In the mid-eighteenth century, Swiss-born mathematician Leonhard Euler developed a formula so innovative and complex that it continues to inspire research, discussion, and even the occasional limerick. Dr. Euler's fabulous Formula shares the fascinating story of this groundbreaking formula - long regarded as the gold standard for mathematical beauty - and shows why it still lies at the heart of complex number theory.

This book is the sequel to Paul Nahin's *An Imaginary Tale: The Story of  $e^{i\pi} + 1 = 0$* , which chronicled the events leading up to the discovery of one of mathematics' most elusive numbers, the square root of minus one. Unlike the earlier book, which devoted a significant amount of space to the historical development of complex numbers, Dr. Euler begins with discussions of many sophisticated applications of complex numbers in pure and applied mathematics, and to electronic technology. The topics covered span a huge range, from a never-before-told tale of an encounter between the famous mathematician G. H. Hardy and the physicist Arthur Schuster, to a discussion of the theoretical basis for single-sideband AM radio, to the design of chase-and-escape problems.

The book is accessible to any reader with the equivalent of the first two years of college mathematics (calculus and differential equations), and it promises to inspire new applications for years to come. Or as Nahin writes in the book's preface: To mathematicians ten thousand years hence, 'Euler's formula will still be beautiful and stunning and untarnished by time.'

## Inhalt

\*What This Book Is About, What You Need to Know to Read It, and WHY You Should Read It - 'When Did Math Become Sexy?' concept of mathematical beauty - equations, identities, and theorems - mathematical ugliness -

beauty redux

Complex Numbers (an assortment of essays beyond the elementary involving complex numbers): The 'mystery' of  $e^{i\pi}$  - The Cayley-Hamilton and De Moivre theorems - Ramanujan sums a series - Rotating vectors and negative frequencies - The Cauchy-Schwarz inequality and falling rocks - Regular  $n$ -gons and primes - Fermat's last theorem, and factoring complex numbers - Dirichlet's discontinuous integral

Vector Trips (some complex plane problems in which direction matters): The generalized harmonic walk - Birds flying in the wind - Parallel races - Cat-and-mouse pursuit - Solution to the running dog problem

The Irrationality of  $\pi$  ("higher" math at the sophomore level) : *The irrationality of  $D$  - The  $R(x) = B(x)ex + A(x)e$  equation,  $D$ -operators, inverse operators, and operator commutativity - Solving for  $A(x)$  and  $B(x)$  - The value of  $R(Di)$  - The last step (at last!)*

*Fourier Series (named after Fourier but Euler was there first - but he was, alas, partially WRONG) - Functions, vibrating strings, and the wave equation - Periodic functions and Euler's sum - Fourier's theorem for periodic functions and Parseval's theorem - Discontinuous functions, the Dirichlet's evaluation of Gauss's quadratic sum - Hurwitz and the isoperimetric inequality - Fourier Integrals (what happens as the period of a periodic function becomes infinite, and the neat Dirac's impulse function) - Fourier's integral theorem - Rayleigh's energy formula, convolution, Some curious spectra - Poisson summation - Reciprocal spreading and the uncertainty principle - Hardy and Schuster, and their optical integral*

*Electronics and  $e^{-1}$  (technological applications of complex numbers that Euler, who was a practical man) - Why this chapter is in this book - Linear, time-invariant systems, convolution (again), transfer functions - The modulation theorem, synchronous radio receivers, and how to make a speech scrambler - The sampling theorem, and multiplying by sampling and filtering - More neat tricks with Fourier transforms - Single-sided transforms, the analytic signal, and single-sided band radio*

## Author Info

Paul J. NAHIN is Professor Emeritus of Electrical Engineering at the University of New Hampshire. He is the author of *Duelling Idiots and Other Probability Puzzlers*, *When Least Is Best: How Mathematicians Discovered Many Clever Ways to Make Things as Small (or as Large) as Possible*, and *An Imaginary Tale: The Story of  $e^{i\pi}$*  (all Princeton). He and his wife Pat live with three enormous tabby cats in a country cape in Lee, New Hampshire.